IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/MATSD/SP1/ENG/TZ0/XX+

22067401

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Wednesday 3 May 2006 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

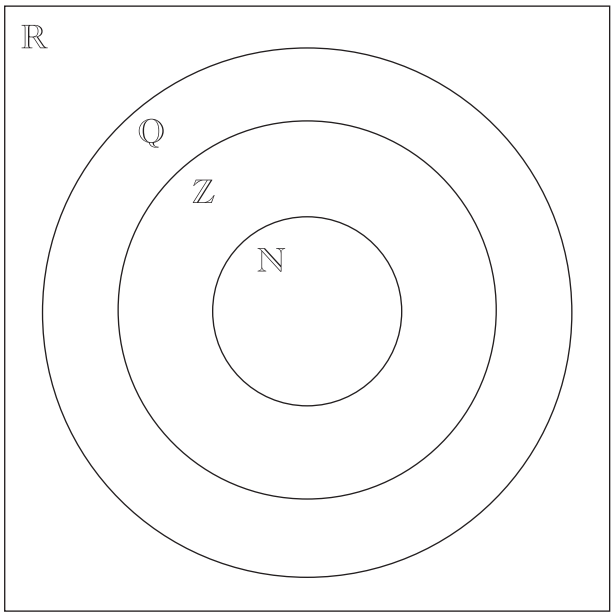
Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The Venn diagram below shows the universal set of real numbers $\mathbb{R}$ and some of its important subsets:

$\mathbb{Q}$: the rational numbers,
$\mathbb{Z}$: the integers,
$\mathbb{N}$: the natural numbers.

Write the following numbers in the correct position in the diagram.

$$-1, 1, \pi, \frac{7}{16}, 3.333\dot{3}, \sqrt{3}.$$



Working:

2. The heights (cm) of seedlings in a sample are shown below.

6	3,	7			
7	2,	5,	8		
8	3,	6,	6,	8,	8
9	2,	5,	7,	8	
10	3,	6,	6		
11	2,	2			

key 6 | 3 represents 63 cm

(a) State how many seedlings are in the sample.

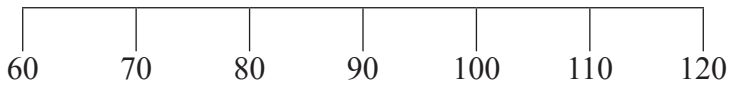
(b) Write down the values of

(i) the median,

(ii) the first and third quartile.

(c) Calculate the range.

(d) Using the scale below, draw a box and whisker plot for this data.



Working:

Answers:

(a) _____

(b) (i) _____

(ii) _____

(c) _____

3. A problem has an **exact** answer of $x = 0.1265$.

(a) Write down the **exact** value of x in the form $a \times 10^k$ where k is an integer and $1 \leq a < 10$.

(b) State the value of x given correct to **two** significant figures.

(c) Calculate the percentage error if x is given correct to **two** significant figures.

Working:

Answers:

(a) _____

(b) _____

(c) _____

4. Consider the statements

p : The sun is shining.
q : I am wearing my hat.

(a) Write down, in words, the meaning of $q \Rightarrow \neg p$.

(b) Complete the truth table.

p	q	$\neg p$	$q \Rightarrow \neg p$
T	T		
T	F		
F	T		
F	F		

(c) Write down, in symbols, the converse of $q \Rightarrow \neg p$.

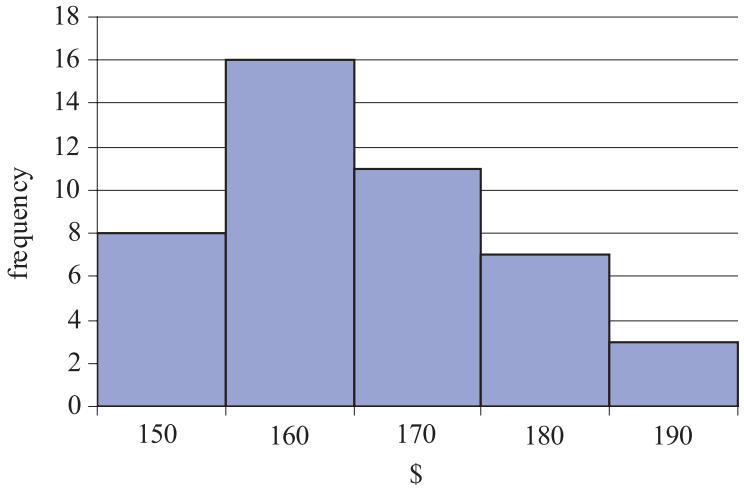
Working:

Answers:

(a) _____

(c) _____

5. The histogram below shows the amount of money spent on food each week by 45 families. **The amounts have been rounded to the nearest 10 dollars.**



(a) Calculate the mean amount spent on food by the 45 families.

(b) Find the **largest possible amount** spent on food by a single family in the **modal** group.

(c) State which of the following amounts could **not** be the total spent by all families in the modal group:

(i) $ 2430 (ii) $ 2495 (iii) $ 2500 (iv) $ 2520 (v) $ 2600

Working:

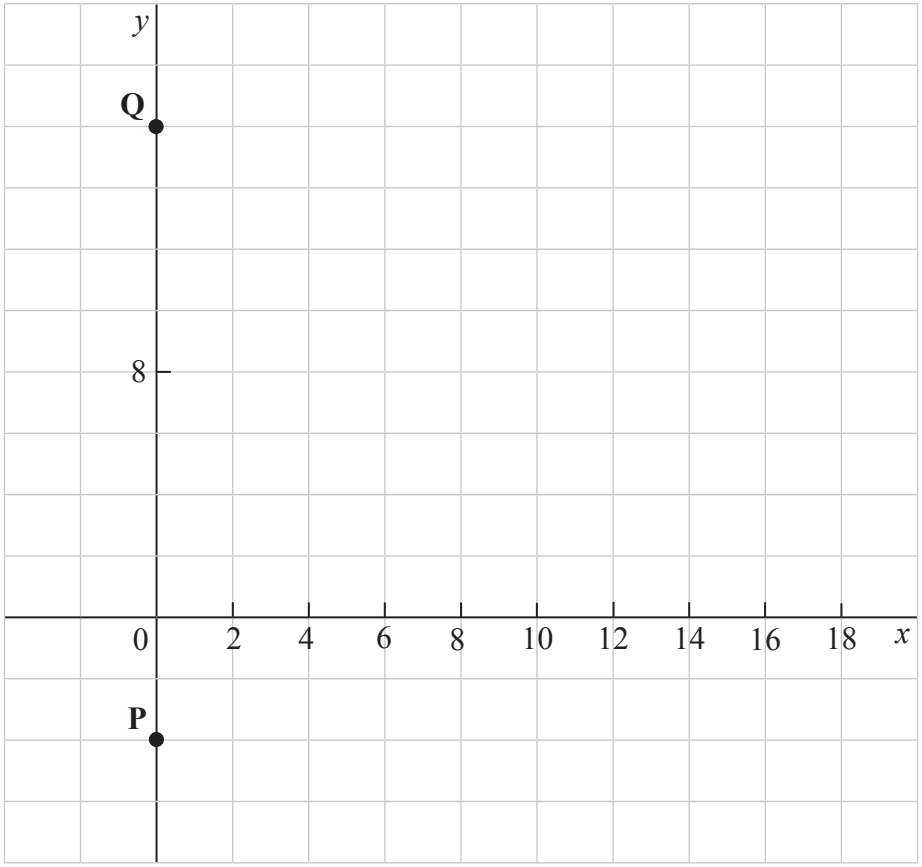
Answers:

(a) _____

(b) _____

(c) _____

6. Points $P(0, -4), Q(0, 16)$ are shown on the diagram.



(a) Plot the point $R(11, 16)$.

(b) Calculate angle QPR.

M is a point on the line PR. M is 9 units from P.

(c) Calculate the area of triangle PQM.

Working:

Answers:

(b) _____

(c) _____

7. The function $Q(t) = 0.003t^2 - 0.625t + 25$ represents the amount of energy in a battery after t minutes of use.

(a) State the amount of energy held by the battery immediately before it was used.

(b) Calculate the amount of energy available after 20 minutes.

(c) Given that $Q(10) = 19.05$, find the average amount of energy produced per minute for the interval $10 \leq t \leq 20$.

(d) Calculate the number of minutes it takes for the energy to reach zero.

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

8. In a group of fifteen students, three names begin with the letter B and four begin with a G. The remaining eight names begin with A, C, D, E, F, H, I and J respectively.

The 15 names are placed in a box. The box is shaken and two names are drawn out.

Find the probability that

(a) both names begin with any letter except G or B,

(b) both names begin with the same letter,

(c) both names begin with the letter H.

Working:

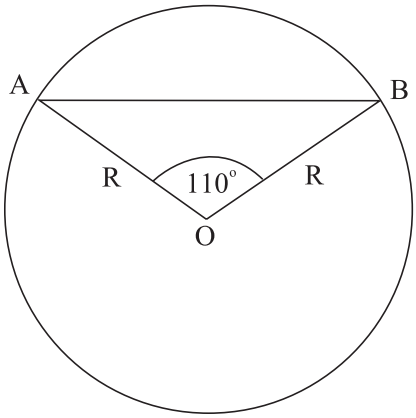
Answers:

(a) _____

(b) _____

(c) _____

9. The diagram shows a circle of radius R and centre O. A triangle AOB is drawn inside the circle. The vertices of the triangle are at the centre, O, and at two points A and B on the circumference. Angle AOB is 110 degrees.



(a) Given that the area of the circle is $36\pi\ \text{cm}^2$, calculate the length of the radius R.

(b) Calculate the length AB.

(c) Write down the side length L of a square which has the same area as the given circle.

Working:

Answers:

(a) _____

(b) _____

(c) _____

10. Sven is travelling to Europe. He withdraws $ 800 from his savings and converts it to euros. The local bank is buying euros at $ 1: € 0.785 and selling euros at $ 1: € 0.766.

(a) Use the appropriate rate above to calculate the amount of euros Sven will receive.

(b) Suppose the trip is cancelled. How much will he receive if the euros in part (a) are changed back to dollars?

(c) How much has Sven lost after the two transactions? Express your answer as a percentage of Sven's original $ 800.

Working:

Answers:

(a) _____

(b) _____

(c) _____

11. Jacques can buy six CDs and three video cassettes for $ 163.17 or he can buy nine CDs and two video cassettes for $ 200.53.

(a) Express the above information using two equations relating the price of CDs and the price of video cassettes.

(b) Find the price of one video cassette.

(c) If Jacques has $ 180 to spend, find the exact amount of change he will receive if he buys nine CDs.

Working:

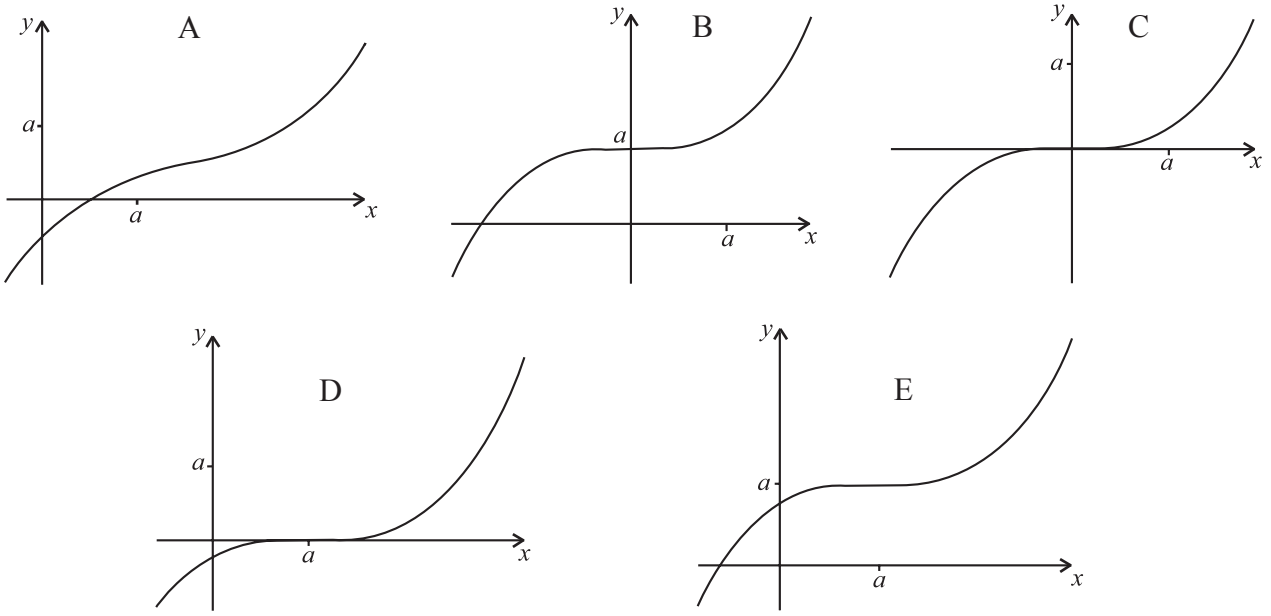
Answers:

(a) _____

(b) _____

(c) _____

12. The following figures show the graphs of $y = f(x)$ with $f(x)$ chosen to be various cubic functions. The value of a is positive.



(a) In the table below, write the letter corresponding to the graph of $y = f(x)$ in the space next to the cubic function.
(Note: one of the graphs is **not** represented in this table)

cubic function $f(x)$	graph label
$f(x) = x^3 + a$	
$f(x) = (x-a)^3 + a$	
$f(x) = x^3$	
$f(x) = (x-a)^3$	

(b) State which **one** of the graphs represents a function that has a positive gradient for **every** value of x.

(c) State how many of the graphs have the x-axis as a tangent at some point.

Working:

Answers:

(b) _____

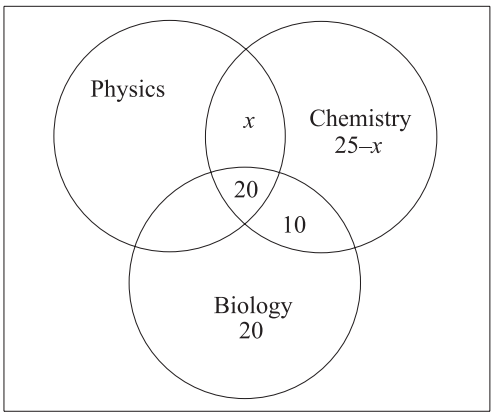
(c) _____

13. At a certain school there are 90 students studying for their IB diploma. They are required to study **at least one** of the subjects: Physics, Biology or Chemistry.

50 students are studying Physics,
60 students are studying Biology,
55 students are studying Chemistry,
30 students are studying both Physics and Biology,
10 students are studying both Biology and Chemistry but not Physics,
20 students are studying all three subjects.

Let x represent the number of students who study both Physics and Chemistry but not Biology. Then $25–x$ is the number who study Chemistry only.

The figure below shows some of this information and can be used for working.



U with $n(U) = 90$

(a) Express the number of students who study Physics only, in terms of x.

(b) Find x.

(c) Determine the number of students studying **at least two** of the subjects.

Working:

Answers:

(a) _____

(b) _____

(c) _____

14. A function $f(x)$ is defined by $f(x) = 3x^4 + \frac{2}{x} - \frac{x}{4} + 1$, $(x \neq 0)$.

(a) Calculate the 2nd derivative $f''(x)$.

(b) Find the value of $f''(x)$ at the point $\left(1, \frac{23}{4}\right)$.

Working:

Answers:

(a) _____

(b) _____

15. Two functions $f(x)$ and $g(x)$ are given by

$$f(x) = \frac{1}{x^2 + 1},$$
$$g(x) = \sqrt{x},\ x \geq 0.$$

(a) Sketch the graphs of $f(x)$ and $g(x)$ together on the same diagram using values of x between –3 and 3, and values of y between 0 and 2. You must label each curve.

(b) State how many solutions exist for the equation $\frac{1}{x^2 + 1} - \sqrt{x} = 0$.

(c) Find a solution of the equation given in part (b).

Working:

Answers:

(b) _____

(c) _____